Funnel Volumes: Volume and Weight

Water drains from an open funnel. How is the volume of water changing with time? At what rate is the water level decreasing? How long will it take for the funnel to drain completely? These are a few questions that can be investigated by collecting weight data for a draining funnel, and then developing a mathematical model to describe the data.

In this activity, you will use a force sensor to measure the weight of a water-filled funnel as it drains. The weight versus time data can be graphed and analyzed using statistical features.

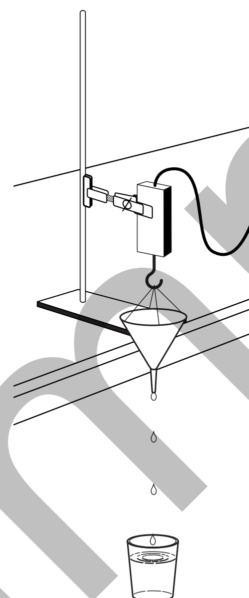

Figure 1

OBJECTIVES

- Record weight versus time data for a draining funnel.
- Describe the data using concepts of intercept and slope of a linear function.

MATERIALS

TI-Nspire handheld **or** computer and TI-Nspire software	funnel
data-collection interface	string
Vernier Force Sensor	bucket
	water

PROCEDURE

1. Set the range switch on the Force Sensor to 10 N. Connect the Force Sensor to the data-collection interface. Connect the interface to the TI-Nspire handheld or computer.

2. Choose New Experiment from the Experiment menu to set up DataQuest for data collection.

3. Arrange the Force Sensor and funnel for data collection (see Figure 1).

4. Place a cup or bucket on the floor to catch water as it drains from the funnel.

5. Block the funnel hole with your finger and fill the funnel completely with water. How long will it take for the funnel to drain? Remove your finger from the funnel hole and estimate the time required for the funnel to empty. Make a note of the time in seconds; you will need this value later.

6. If the drain time for your funnel is more than three seconds, you will use the default data-collection rate (The number of points collected should be 501); skip to Step 7. If the estimated time to drain the funnel is less than three seconds, adjust the data collection time as described below.
   a. Choose Collection Setup from the Experiment menu.
   b. Enter **5** as the duration (seconds). The number of points collected should be 251.
   c. Select OK.

7. In this activity you only want to measure the weight of the water in the funnel. To do this, you need to set the force scale so that the Force Sensor reads zero when supporting the weight of the empty funnel and string.
   a. Hold the sensor so that the funnel can hang downward from the hook. Place the bucket under the funnel. Allow the funnel to stop swinging.
   b. Ensure that the funnel is not swinging and choose Set Up Sensors ► Zero from the Experiment menu. This will set the current weight reading to zero.

8. You are now ready to collect weight versus time data.
   a. Block the funnel hole with a finger, and fill the funnel completely with water.
   b. Start data collection ().
   c. When the first data point appears on the screen, remove your finger from the funnel and allow it to empty. Take care to not touch the funnel while it is emptying, and make sure that it does not swing.
   d. After data collection is complete you will see a graph of the weight of the water in the funnel versus time.

9. Your graph should be a uniformly decreasing function, with a horizontal section at the end. If you need to repeat data collection, repeat Step 8.

10. Since data collection probably did not exactly coincide with the emptying of the funnel, you need to remove the data taken before you released the water and after the funnel was empty.
    a. Select the smoothly decreasing portion of the graph.
    b. Choose Strike Data ► Outside Selected Region from the Data menu.
    c. The data outside the region you just marked will be removed.
    d. Choose Autoscale Now from the Graph menu.

DATA TABLE

x_1		y_1	
x_2		y_2	
slope			
x-intercept		*y*-intercept	

ANALYSIS

1. Click any point on the graph. Use ► and ◄ to trace across the graph. Choose two well-separated points on the graph. Record their x and y coordinates in your data table.

2. Use the two points to find the slope of the weight *vs.* time graph. Record the slope, with units, in the data table.

   Answer Analysis Question 1.

3. You still need to find the *y*-intercept of this segment, but you can't just trace to the *y* axis because the data doesn't necessarily go that far. However, using the definition of slope we can write down the point-slope form of a line, or $y - y_1 = m(x - x_1)$. Here *y* and *x* are variables, *m* the slope, and x_1 and y_1 are the values of a point on the line. Use this relation to answer Analysis Question 2.

4. Create a Graphs page so you can model the graph with a linear function.
   a. Insert a Graphs page.
   b. Insert the Sensor Console in order to input the graph from DataQuest. Verify that your data appears and then close the Sensor Console.
   c. Choose Zoom – Data from the Window/Zoom menu to view all of your data.
   d. Chose Function from the Graph Type menu.
   e. Enter your model equation into the Entry Line replacing *m* and *b* with the values you determined earlier.

$$f1(x) = mx + b$$

5. Experiment with your movable line to find the best value for *m* and *b* by grasping the line to translate and rotate it.

   Answer Analysis Question 3.

6. Determine the *x* intercept for the linear equation you have found. Record the value in your data table, including units.

   Answer Analysis Question 4.

7. You can have DataQuest perform a linear regression, a type of least-squares regression, on all the data points.
   a. Return to your DataQuest page.
   b. Choose Curve Fit ► Linear from the Analyze menu.
   c. Use the parameters *m* and *b* to answer Analysis Questions 5–7.
   d. (optional) Print your graph.

ANALYSIS QUESTIONS

1. Why is the value of the slope negative?
2. Write the equation of the line fitting the data in the traditional $y = mx + b$ form. Record the y-intercept in your data table.
3. Is your line a good fit for the data? Does the line pass directly through any particular points? Why?
4. What is the physical interpretation of the x-intercept?
5. Use the parameters m and b that appear on the screen to write the equation of the best-fit line.
6. Is the line suggested by the regression consistent with your results you calculated above? Why might they be different?
7. What physical characteristics of the funnel could be changed to reduce the rate at which the volume of water in the funnel is changing with time? What would happen to the slope of the fitted line to data taken with the modified funnel?

EXTENSION

Determine the radius-to-height ratio for the funnel you used. Write an equation relating volume, V and height, h. Differentiate this equation with respect to time. Use your weight versus time graph to find dW/dt. Convert this to dV/dt using the specific gravity of water. Substitute this value into the differential equation. Solve for h as a function of t.

Use this relationship to determine when the water level will be at half its initial value. To test your answer, plug the hole and re-fill the funnel. Unplug the hole and activate a stopwatch simultaneously. Re-plug the hole when the stopwatch reading matches the time you computed above. Is the height of the water in the funnel approximately half its initial value?

Activity
4

TEACHER INFORMATION

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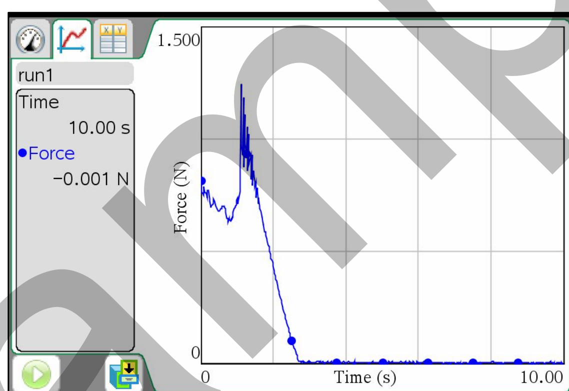
1. The student pages with complete instructions for data collection using DataQuest (TI-Nspire Technology), EasyData (TI-83/84 Plus calculators), DataMate (other TI calculators), or Logger *Pro* software can be found on the CD that accompanies this book. See *Appendix A* for more information.

2. The Force Sensor reads in units of Newtons, the SI unit of force. Weight is measured in Newtons. The weight of the water remaining in the funnel is thus measured in Newtons. If you want to convert to cm^3 of water, multiply the force readings by 102 cm^3/N.

3. Hold the probe and funnel so that both are level while data is being collected. Be careful not to touch the funnel during data collection.

SAMPLE RESULTS



Sample data

DATA TABLE

x_1	1.6 s	y_1	0.74 N
x_2	2.64 s	y_2	0.046 N
slope	–0.67 N/s		
x-intercept	2.66 s	y-intercept	1.82 N

ANSWERS TO ANALYSIS QUESTIONS

1. The slope is negative because the weight of the water-filled funnel is decreasing with time.

2. Answers may vary.

3. Yes, the line is a good fit. The fitted line passes directly through the point used in the point-slope formula.

4. The x intercept is the time the funnel was first empty.

5. $y = -0.66x + 1.76$.

6. The new line is nearly the same as the one found using two points, but could be closer to the data as a whole since the whole line is used to determine the slope and intercept.

7. Making the opening smaller would slow the release of the water, so that the rate of change of the volume of water would be smaller. The slope of the fitted line would then be smaller in magnitude.